Synthetic Forward and Put-Call-Parity

A synthetic long forward can be created by purchasing a call option and writing a put option with the same strike price and the same expiration date.

Let's say, that an investor buys a $500-strike call option and sells a $500-strike put option, both options have the same expiration time T. At T, the spot price will be equal to, above or below the strike price.

If the spot price is above the strike price K, the investor will exercise the call option and buy the underlier for $500. The put will not be exercised. If the spot price is below the strike price the written put will be exercised against the investor, forcing him to buy the underlier at $500. The call will expire worthless. In either case, the underlier will be purchased at the strike price.

The payoff of this combined position is

$$\underbrace{\max\{0, S - K\}}_{\text{Call}} - \underbrace{\max\{0, K - S\}}_{\text{Put}} = S - K$$

but this is actually the payoff of a long forward contract expiring at time T and with a forward price of $500.

Be aware of the differences between an original long forward and a synthetic long forward: (i) the original long forward has no upfront premium payment while the synthetic long forward requires a net option premium (= abs[Call(K,T) – Put(K,T)]) as upfront payment. (ii) At expiration at time T we pay the forward price with the original long forward, whereas with the synthetic long forward we pay the strike price.

Let $F_{0,T}$ denote the "no-arbitrage" forward price. This means you have to pay $0 today and at time T you are obliged to buy the asset at $F_{0,T}$.

The cost of the contract today is the present value of $F_{0,T}$, PV($F_{0,T}$). Now we pay Call(K, T) – Put(K, T) today to buy the option on that asset for K at time T which results in a cost at time 0 of Call(K, T) – Put(K, T) + PV(K). And using the "no-arbitrage" pricing, the net cost of the asset must be the same whether through options or forward contract, that is

$$\text{Call}(K, T) - \text{Put}(K, T) = PV(F_{0,T} - K). \qquad \text{(CP.1)}$$

That means:
{The premium for a Call Option (at strike price K and expiration at time T)
minus
the premium for a Put Option (at strike price K and expiration at time T)}
equals
{the present value of the difference between the "no-arbitrage" forward price (with the same expiration time T) and the option's strike price.}

Let us have a closer look at the different terms:

$PV(F_{0,T})$

is equal to the current spot price of the underlier, and therefore represents a long position in the underlier.

PV(K)

is the present value of an amount K payable in T years; it represents a T-year-zero-coupon bond with maturity value K.

Let us show the Put-Call-Parity with the help of the following graphic:

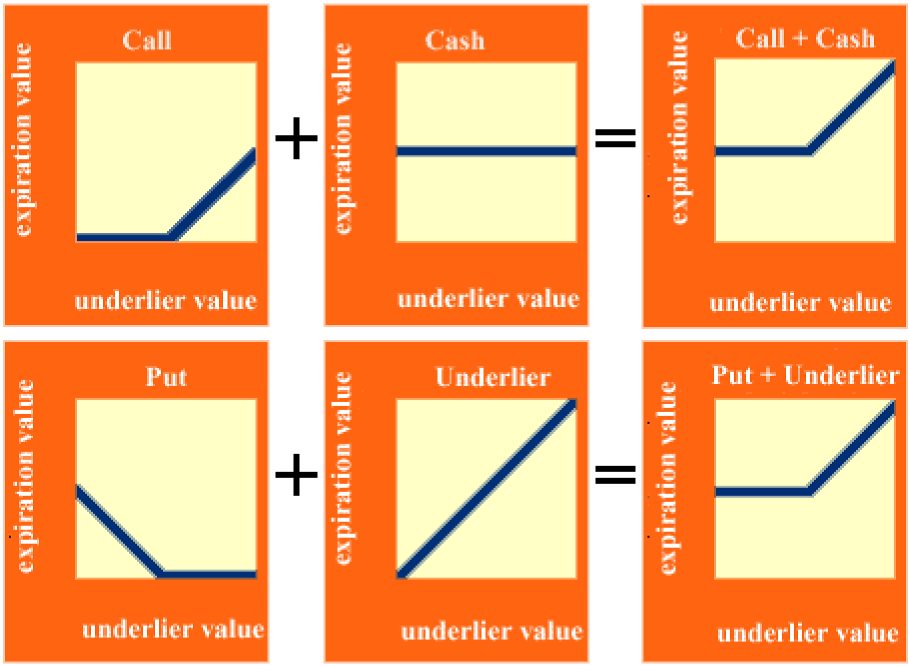

Exhibit CP.1: Put-Call-Parity (**Kind Courtesy of Glyn Holton** – www.riskglossary.com)

We have two portfolios: One comprises a Call Option and a cash amount (the PV of the call options strike price), and the other portfolio comprises a Put Option and the underlier. When the expiration value of each of these portfolios is the same, then the "no-arbitrage" opportunity tells us that the present value of these expiration values must also be the same. And this is called **Put-Call-Parity.**

Let's refer to formula (CP.1) and make some rearrangements:

$PV(F_{0,T})$ – Call(K, T) = - Put(K, T) + PV(K)
Writing a "covered call" is equivalent to writing a put option and buying a zero-coupon-bond for the amount K

$-PV(F_{0,T})$ – Put(K, T) = - Call(K, T) – PV(K)   Writing a "covered put" is equivalent to writing a call option and taking out a loan of amount K.